

4-3-2013

Cylindrical Co-ordinates

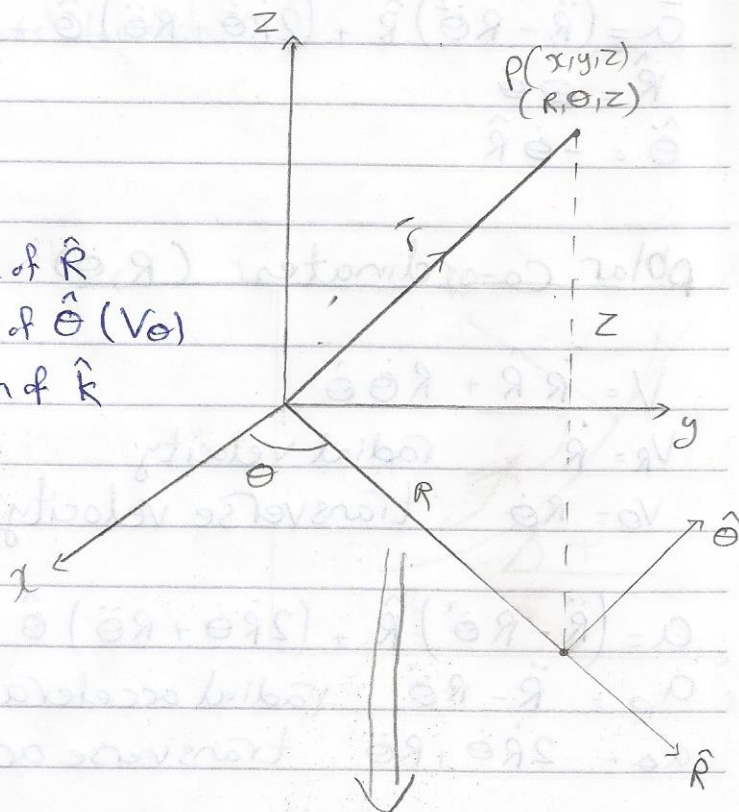
$$\vec{r} = R\hat{R} + z\hat{k}$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \dot{R}\hat{R} + R\dot{\hat{R}} + \dot{z}\hat{k}$$

$\dot{R}\hat{R} \Rightarrow v_R \Rightarrow$ Velocity in direction of \hat{R}

$R\dot{\hat{R}} = R\dot{\theta}\hat{\theta} \Rightarrow$ Velocity in direction of $\hat{\theta}$ (v_θ)

$\dot{z}\hat{k} \Rightarrow v_z \Rightarrow$ Velocity in direction of \hat{k}



$$\hat{R} = \cos\theta\hat{i} + \sin\theta\hat{j}, \quad \hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

$$\dot{\hat{R}} = -\sin\theta(\dot{\theta})\hat{i} + \cos\theta(\dot{\theta})\hat{j}$$

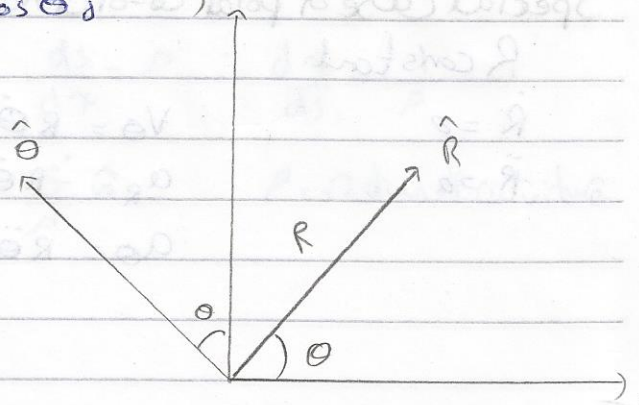
$$= \dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$\dot{\hat{R}} = \dot{\theta}\hat{\theta}$$

$$\dot{\hat{\theta}} = -\cos\theta(\dot{\theta})\hat{i} - \sin\theta(\dot{\theta})\hat{j}$$

$$= -\dot{\theta}(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\dot{\hat{\theta}} = -\dot{\theta}\hat{R}$$



$$\vec{v} = \dot{R}\hat{R} + R\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$$

$$\frac{d\vec{v}}{dt} = \vec{a} = \ddot{R}\hat{R} + \dot{R}\dot{\hat{R}} + \dot{R}\dot{\theta}\hat{\theta} + R\ddot{\theta}\hat{\theta} + R\dot{\theta}\dot{\hat{\theta}} + R\dot{\theta}\dot{\hat{\theta}} + \ddot{z}\hat{k}$$

$\dot{\theta}\hat{\theta}$

$-\dot{\theta}\hat{R}$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta} + \ddot{z}\hat{k}$$

Summary

$$\vec{r} = R\hat{r} + z\hat{k}$$

$$\vec{v} = \dot{R}\hat{r} + R\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{r} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta} + \ddot{z}\hat{k}$$

$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$$

$$\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

polar co-ordinates (R, θ)

$$\vec{v} = \dot{R}\hat{r} + R\dot{\theta}\hat{\theta}$$

$$v_R = \dot{R} \quad \text{radial velocity}$$

$$v_\theta = R\dot{\theta} \quad \text{transverse velocity}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{r} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta}$$

$$a_R = \ddot{R} - R\dot{\theta}^2 \quad \text{radial acceleration}$$

$$a_\theta = 2\dot{R}\dot{\theta} + R\ddot{\theta} \quad \text{transverse acceleration}$$

Special case of polar co-ordinates: circular motion

R constant

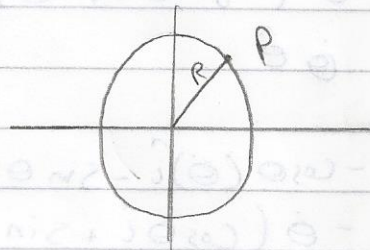
$$\dot{R} = 0$$

$$\ddot{R} = 0$$

$$v_\theta = R\dot{\theta}$$

$$a_R = -R\dot{\theta}^2, \quad \vec{a} = -\dot{\theta}^2\vec{r}$$

$$a_\theta = R\ddot{\theta}$$



Ex. 11.12

$$\theta = 0.15t^2, \quad r = 0.9 - 0.12t^2 \quad \text{at } \theta = 30^\circ \quad OA = 0.9 \text{ m}$$

$$a) \quad \dot{r} = -0.24t, \quad \ddot{r} = -0.24$$

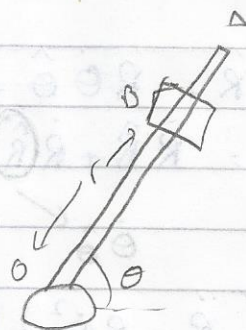
$$\dot{\theta} = 0.3t, \quad \ddot{\theta} = 0.3$$

$$\theta = 30^\circ = \frac{\pi}{6} \quad \frac{\pi}{6} = 0.15t^2 \quad t = 1.87$$

$$v = \sqrt{v_R^2 + v_\theta^2}$$

$$= \sqrt{(\dot{r})^2 + (R\dot{\theta})^2} = \sqrt{(-0.24 \times 1.87)^2 + (0.3 \times 1.87 \times 0.48)^2}$$

$$= 0.52$$



$$b) \bar{a} = \ddot{R} - R\dot{\theta}^2$$

$$b) a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-0.24 - 0.48 \times 0.56^2)^2 + (0.48 \times 0.3 + 2 \times -0.449 \times 0.56)^2}$$

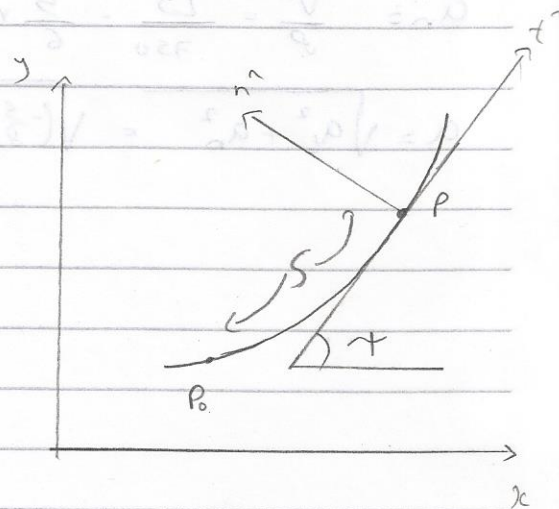
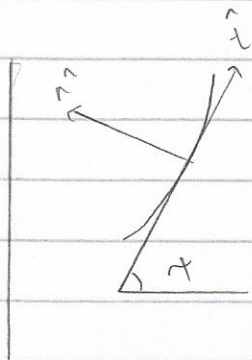
$$= 0.531 \text{ m/s}^2$$

$$c) a_{B/OA} = -0.24 \text{ m/s}^2 \quad (0.24 \text{ m/s}^2 \text{ towards } O)$$

Intrinsic Co-ordinates

$$\bar{v} = v\hat{t} = \dot{s}\hat{t}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \ddot{s}\hat{t} + \dot{s}\hat{t}'$$



$$\hat{t}' = \gamma \hat{n} \quad (\text{same as } \hat{R}' = \dot{\theta} \hat{\theta})$$

$$\gamma = \frac{d\gamma}{dt} = \frac{d\gamma}{ds} \cdot \frac{ds}{dt}$$

$$\frac{ds}{d\gamma} = \rho \quad \therefore \frac{d\gamma}{ds} = \frac{1}{\rho}$$

$$\frac{ds}{dt} = \dot{s}$$

$$\gamma = \frac{\dot{s}}{\rho}$$

$$\hat{t}' = \frac{\dot{s}}{\rho} \hat{n}$$

$\rho = \text{radius of curvature}$

$$\bar{a} = \ddot{s}\hat{t} + \dot{s}\hat{t}'$$

$$= \ddot{s}\hat{t} + \frac{\dot{s}^2}{\rho}\hat{n}$$

$\ddot{s} \rightarrow \text{tangential acceleration } a_t$

$\frac{\dot{s}^2}{\rho} \rightarrow \text{normal acceleration } a_n$

Ex 11.10

$$90 \text{ km/h} = 25 \text{ m/s}$$

$$72 \text{ km/h} = 20 \text{ m/s}$$

$$\vec{a} = a_t \hat{t} + a_n \hat{n} = \dot{v} \hat{t} + \frac{v^2}{\rho} \hat{n}$$

$$a_t = \frac{v-u}{t} = \frac{20-25}{8} = -\frac{5}{8} \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{750} = \frac{5}{6} \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-\frac{5}{8}\right)^2 + \left(\frac{5}{6}\right)^2} = 1.04$$

